

# Dynamic Coalition Formation in a Smart Grid: A Game Theoretic Approach

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**Abstract**—In this paper, the problem of optimal energy distribution by dynamically changing the size of coalition, which consists of one micro-grid and several customers, is studied using the theory of Markov Decision Process (MDP) — a discrete optimization method. In this paper, the micro-grid, which acts as one of the players, needs to decide the size of the coalition for utilizing the generated energy optimally. On the other hand, the customer, which acts as another player, needs to decide its strategies, so as to optimize a trade-off between the associated cost, i.e., communication cost and energy distribution cost, and effective power supply. Using MDP, it is shown how dynamically coalition can be formed and the customer can be assured of an efficient power distribution.

**Index Terms** — Power economy revenue, Smart Grid, customer, game theory, coverage, coalition, energy exchange.

## I. INTRODUCTION

Due to the growing concerns for energy conservation and environment, smart grid [1] has been visualized to be a cyber-physical system that can augment the efficiency, reliability, and robustness of power and energy of the grid by integrating advanced techniques such as PHEVs and Advanced Metering Infrastructure (AMI). In existing power systems, electricity is delivered to the customer by a main electrical grid that delivers power over the low-voltage distribution network. In the presence of micro-grids, it is desirable to allow the micro-grids to service some small geographical areas or group of customers based on their demand, so as to relieve the demand on the main grid [2].

Let us consider a distribution network composing of one substation, which is connected to the main grid, as well as to  $M$  micro-grids in the set  $N$ , where  $M \in N$ . Each micro-grid  $i \in N$  services a certain demand and the difference between its generation and demand is represented as  $Q_i$ . At a given period of time, depending on the customers' demand and generated power, a certain micro-grid  $i \in N$  may have either a surplus of power ( $Q_i > 0$ ) to sell or a need to acquire power to meet its demand ( $Q_i < 0$ ). Therefore, some cooperation exists between the micro-grid and the customer [3].

In this paper, we introduce a suitable cooperation scheme that can help in dynamic coalition formation for proper utilization of energy. The coalition is considered to be consisting of one micro-grid and the customers to whom that micro-grid can distribute energy optimally. Inside the coalition, the micro-

grid can transfer power locally to its customers, but unable to transfer energy directly to the other coalition. Therefore, we propose to form dynamic coalition, so that energy could be distributed properly with less amount of energy loss.

The rest of the paper is organized as follows. We briefly present the related literature in Section II. Section III describes the system model. In Section IV, we formulate the stochastic optimization method using MDP [4]–[6], and, thereafter, we discuss its properties. We also propose a distributed algorithm and discuss the performance of the algorithm in Section V. Finally, we conclude the paper while citing few research directions, in Section VI.

## II. RELATED WORKS

In the last few years, lot of research work on smart grid emerged, viz., [7]–[13]. The early works considered that each micro-grid  $i \in N$  exchanges the amount of power  $Q_i$  with the main grid using the main substation, in the absence of storage and cooperation [3]. The transfer of power is accompanied by power loss over the distribution lines inside the micro-grid network. The total loss over the distribution line due to power transfer is given by [3]:

$$L(i) = -w_i P_i^{loss} \quad (1)$$

In Equation (1),  $P_i^{loss}$  is the power lost during power exchange between  $i$  and the substation, and  $w_i$  is the price paid by  $i$  per unit of power loss. The power loss,  $P_i^{loss}$ , is a function of several factors such as the distance between the micro-grid and the substation (due to resistance), the power  $Q_i$  that is being transferred, as well as the losses at the transformers of the substation.

In [2], the authors proposed a method for coalition formation in which a coalition consists of multiple micro-grids. In such a design the customer had no choice to change the coalition. One grid having excess energy can transfer that amount of energy to another grid, which is energy deficient [2]. In that scenario, there will be some loss of energy due to local transfer of energy. Mathematically, the loss can be expressed as [2]:

$$Loss(s_i, s_j) = \sum_{i \neq j} w_{ij} P_{ij}^{loss} \quad (2)$$

Further, it may be stated that although [2] and [3] considered the concept of coalition games in the context of smart grid, our work differs from them in that we use coalition game theory to form the coalition themselves. On the other hand, the above mentioned existing works use the theory to distribute energy between grids.

In [14], the authors proposed a distributed load management scheme. They assumed that the customers know their energy usages and can schedule their consumption priority according to the new pricing policy. The customers make their own bid by broadcasting their energy demand vector when a new customer is included in the network. They proposed an algorithm to bid the energy vector. Once the convergence reaches the demand distribution, it is fixed and is executed according to defined schedule.

In [13], the authors proposed a non-cooperative game approach for controlling both loads and energy sources in a small-scale power set  $\aleph = L \cup S$ , which represents the group of loads  $L$  and power sources  $S$ , and the strategy of each player depending on its type. The objective functions of the load and the source are application dependent. However, in general, they are functions of the strategies, the currents, the voltages, and the impedance, as discussed in Reference [13].

### III. SYSTEM MODEL

In Figure 1, the schematic view of a typical smart grid is shown. Let us consider a power system consisting of  $S$  substations. Every substation  $k \in S$  consists of  $M_k$  number of macro-grids and there are  $(N_j)_k$  micro-grids under each macro-grid  $j \in M$ . Hence,  $N$  number of coalitions will be formed. Assuming that each coalition  $i \in N$  has an area  $a_i$ ,

$$\left( \sum_{i \in N_j} a_i \right)_j = \alpha_j \quad (3)$$

where,  $N_j$  is the total number of coalitions under the  $j^{th}$  macro-grid and the  $j^{th}$  macro-grid has a total area of  $\alpha_j$ .

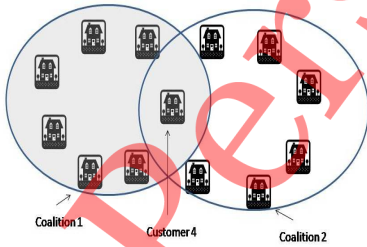


Fig. 1: Smart Grid

We have another equation for macro-grids,

$$\left( \sum_{j \in M} \alpha_j \right)_k = A_k \quad (4)$$

where,  $M_k$  is the total number of macro-grids under the  $k^{th}$  substation and the total area of the  $k^{th}$  substation is  $A_k$ .

We also assume that the  $i^{th}$  micro-grid has a generation capacity of  $G_i$  at a certain time  $t$ . This generated energy,  $G_i$ ,

can be sold to the  $\Phi$  number of customers that are within the  $i^{th}$  coalition, thereby allowing them to meet their demand. The grid will set an appropriate price  $p$  (per unit energy) for selling the generated energy to optimize its power economy revenue.

Each customer  $n \in \Phi$ , where  $\Phi$  is the set of all the customers, will request a certain amount of energy  $x_n$  from the micro-grid, so as to meet its energy requirements. This demand of energy may vary temporally based on different parameters such as the energy storage capacity, the price  $p$  per unit of energy and the nature of usage of energy. Since the net energy generation capacity for the  $i^{th}$  micro-grid is fixed, the demand of customers must satisfy

$$\left( \sum x_n \right)_i \leq G_i \quad (5)$$

where  $\forall i \in N$  and  $\forall n \in \Phi$ .

To successfully complete energy trading, the customers and the micro-grid interact with one another and agree on whether a customer joins a coalition or not. Here, the grid tries to utilize the generated energy properly and increase its power economy revenue. On the other hand, the customer tries to fulfill its total energy requirement efficiently and economically.

### IV. PROPOSED MDP-BASED OPTIMIZATION METHOD

#### A. Game formulation

To formally study the interaction between the grids and the customers, we use MDP [15] to design a multi-level decision making process, as shown in Figure 2, for forming the coalition in a dynamic way. In Figure 2 it is shown how the customer and the grid play games with one another. We consider the customer as Player 1, and the grid as Player 2. Based on the decision of Player 1, the Player 2 chooses its strategy and so on. This game is defined by its strategic form,  $\tau = [(\Phi \cup G), (X_n)_{n \in \Phi}, (U_n)_{n \in \Phi}, p]$ , having the following components:

- i) The customers in  $\Phi$  act as players in the game and respond to the inclusion request by the grids.
- ii) The strategy of each customer  $n \in \Phi$ , which corresponds to the amount of energy  $x_n \in X_n$  from the grid satisfying the constraint  $\sum_{n \in \Phi} x_n \leq G$ .
- iii) The utility function  $U_n$  of each customer  $n$  that captures the benefit of consuming demanded energy  $x_n$ .
- iv) The price  $p$  is the per unit of energy charged by grids.

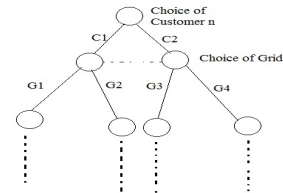


Fig. 2: Multi-level Decision Making Process

**Utility function of a coalition:** For every coalition  $n \in \Phi$ , we define a utility function  $U_n(x_n, x_{-n}, G_i, s_i, a_i, d_n, p)$ , which represents the level of will of a customer to join a coalition. Here,  $G_i$  is the total amount of energy generated by the grid  $i \in N$ , and  $s_i$  is the satisfaction parameter of the  $i^{th}$  grid, which is the measure of satisfaction the grid can achieve by selling energy relative to the generated energy. For example, Grid 1 ( $G_1$ ) and Grid 2 ( $G_2$ ) generate the same amount of energy at a certain point of time, but  $G_1$  is able to sell more energy than  $G_2$ . We can infer that the satisfaction of  $G_1$  is more than the satisfaction of  $G_2$  (i.e.,  $s_2 < s_1$ ). Therefore, the properties that utility of a customer must satisfy are as follows:

i) The utility function of the customers are considered to be non-decreasing, as each customer is interested in consuming more energy. Mathematically,

$$\frac{\delta U_n(x_n, x_{-n}, G_i, s_i, a_i, d_n, p)}{\delta x_n} \geq 0 \quad (6)$$

ii) The marginal benefit of a customer is considered to be a decreasing function, as the satisfaction-level of grid gets saturated as more energy is sold to the customer. Mathematically,

$$\frac{\delta^2 U_n(x_n, x_{-n}, G_i, s_i, a_i, d_n, p)}{\delta x_n^2} < 0 \quad (7)$$

iii) Assuming that each grid generates the same amount of energy, a larger value of  $\sum_{n \in \Phi} x_n$  will lead to higher satisfaction. So, we have,

$$\frac{\delta U_n(x_n, x_{-n}, G_i, s_i, a_i, d_n, p)}{\delta s_i} < 0 \quad (8)$$

iv) The utility function  $U_n$  of a customer is inversely proportional to the radial distance,  $d_n$ , as with the increase in the distance, the delay for communication and power transmission increases. Mathematically,

$$\frac{\delta U_n(x_n, x_{-n}, G_i, s_i, a_i, p)}{\delta d_n} < 0 \quad (9)$$

Therefore, in this work, we consider the following specific utility:

$$U_n(x_n, x_{-n}, G_i, s_i, a_i, p) = G_i x_n + p x_n - \frac{1}{2} s_i x_n^2 - a_i d_n \quad (10)$$

where  $d_n$  is the radial distance of the  $n^{th}$  customer,  $d = \gamma x_n$ , and  $\gamma$  is a constant.  $x_n \in [0, G - \sum_{q=1, q \neq n}^{\Phi} x_q]$  and  $x_{-n} = [x_1, x_2, \dots, x_{n-1}, x_{n+1}, \dots, x_N]$ .

## B. Algorithm

In order to reach the equilibrium in energy distribution from the grid to the customer, the customer and the grid must take their strategy choices with a small communication overhead between one another to form the coalition. In this work, we propose two different algorithms. The customers and the grids individually follow different algorithms. The customer follows its own algorithm to get uninterrupted power supply with less cost per unit, whereas the grid follows its own algorithm to increase its revenue, and tries to utilize the generated energy

properly. By executing the two algorithm sequentially, we infer how dynamically coalition will be formed. First, the grid broadcasts its payoff function to customers and the priority of including any customer to form the new coalition will be based on the radial distance of that customer from the micro-grid. After knowing the payoff function of each grid, those micro-grids, which want to include a particular customer  $n \in N$ , the customer  $n$  will decide whether to accept the proposal of joining the coalition or to decline the proposal based on the consumption of its utility function,  $U_n$ .

**Algorithm for Dynamic Coalition Formation:**

1) **Algorithm for Grids:** Each grid  $i \in N$  calculates its excess energy by evaluating the function  $E(i) = G_i - \sum_{q=1, q \neq n}^{\Phi} x_q$ . A grid broadcasts its payoff function having the amount of excess energy,  $E$ , and the cost per unit,  $p$ . After getting these values, the customer  $n \in \Phi$  makes a decision based on its utility function. In case a customer is unwilling to join the coalition, then the grid receives that information and modifies its previously assigned cost per unit  $p$ , to maximize its revenue. Thereafter, it broadcasts that message. This process continues until the grid makes the proper utilization of its generated energy and gets the maximum revenue by selling the generated energy to the customer. Mathematically,

$$U_i^{g*}(G, \sum_{n=1}^{\Phi} x_n + x_{new}, p^*) \geq U_i^g(G, \sum_{n=1}^{\Phi} x_n, p) \quad (11)$$

In Equation (11),  $U_i^g$  is the utility function of the  $i^{th}$  grid,  $p^*$  is the modified cost per unit energy, and  $p$  is the cost per unit energy prior to the modification.

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### Algorithm 1: Algorithm for Grid

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**Input:** Amount of generated energy  $G_i$  by grid  $i \in N$

**Output:** Request customer  $n \in \Phi$  to join its coalition

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while  $G_i > \sum_{n \in \Phi} x_n$  do
  evaluate  $\sum_{n=1}^{\Phi} x_n$ ;
  if  $(G_i - \sum_{n=1}^{\Phi} x_n) > 0$  then
    evaluate satisfaction factor  $s_i$ , where
       $s_i = \frac{\sum_{n=1}^{\Phi} x_n}{G_i}$ ;
    request a new customer,  $j \notin \Phi$  to join its
    coalition;
  else
    energy generated by grid  $i$ ,  $G_i$ , is properly
    utilized; system is stable, so formed coalition is
    fixed;

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2) **Algorithm for Customers:** Each customer has two choices. One of its options is to join the coalition of the requested grid,  $i$ . Another option is that it will not join that coalition and will remain in the same coalition  $l$ , where  $l \in N$ , and  $l \neq i$ . Before making this choice, the customer calculates its utility function  $U_n(x_n, x_{-n}, G_i, s_i, a_i, p) = G_i x_n + p x_n - \frac{1}{2} s_i x_n^2$ , and chooses the grid having a better utility factor at

that time instant  $t$ , to ensure an uninterrupted power supply in an efficient way. Mathematically,

$$\begin{aligned} U_n^*(x_n, (x_{-n})_i, G_i, s_i, a_i, (d_n)_i, p^*) &\geq \\ U_n(x_n, (x_{-n})_j, G_j, s_j, a_j, (d_n)_j, p) &\end{aligned} \quad (12)$$

where,  $U_n^*$  is the modified utility function of customer  $n$ .

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**Algorithm 2:** Algorithm for Customer

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**Input:** Amount of energy,  $x_n$ , required for customer  $n \in \Phi$

**Output:** Energy requirement of customer  $n \in \Phi$  is fulfilled

**while**  $U_n^* \geq U_n$  **do**  
    evaluate utility function  $U_n$  according to Equation (10);  
    **if**  $(U_n^*(x_n, (x_{-n})_i, G_i, s_i, a_i, (d_n)_i, p^*) \geq U_n(x_n, (x_{-n})_j, G_j, s_j, a_j, (d_n)_j, p))$  **then**  
        | join  $i^{\text{th}}$  coalition,  $i \in N$ ;  
    **else**  
        | remain in the same coalition;

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## V. RESULTS AND DISCUSSIONS

We considered randomly generated positions of the grids and the customers using the MATLAB simulation platform. Based on the distance between a customer and a grid, a matrix is generated. From that matrix, the customer decides to join the coalitions of the grid which has the minimum distance from the customer. In this work, we have considered that the payoff value of all the grids is unity. As the payoff of the grids changes with time, according to that payoff, a customer calculates its utility factor. If the customer gets a higher utility factor for one of the grids, the customer decides to join the coalition of that grid. Thus, we have modified the coalition dynamically and studied its effect on different parameters.

TABLE I: Simulation Parameters

		Coalition 1	Coalition 2
Number of Grids	Scenario 1	50	50
	Scenario 2	2	2
Number of Customers	Scenario 1	100	100
	Scenario 2	10	10
Payoff of each Grid	Scenario 1	1	$\leq 1$
	Scenario 2	1	$\leq 1$

### A. Change in Coalition

In Figure 3, dynamic coalition formation is shown. We have taken two different scenarios. In *Coalition 1* a customer  $n \in \Phi$  chooses a coalition of grid  $i \in N$  as a service provider. But at a certain point of time, the customer  $i$  joins the coalition of grid  $j \in N$ , where  $i \neq j$ , as grid  $j$  provides better consistent energy supply with less cost per unit. Mathematically,

$$U_i^g(G_i, \sum_{k=1}^{\Phi_i} x_k) \leq U_j^g(G_j, \sum_{k=1}^{\Phi_j} x_k) \quad (13)$$

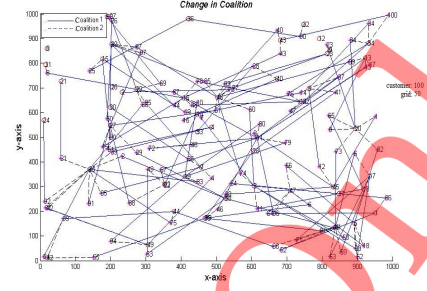


Fig. 3: Dynamical formation of coalition

In Figure 1, we have such a scenario, where one customer,  $x_4$ , has the option to choose two different coalitions. However, depending on its utility function,  $U_4$ , it decides to join Coalition 1 over Coalition 2.

The results for Scenarios 1 and 2 are shown in Figures 3 and 4 respectively.

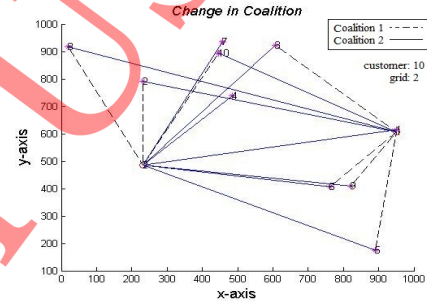


Fig. 4: Dynamical formation of coalition

### B. Utilization of Energy

In Figure 5, the satisfaction factor,  $s_i$ , of the  $i^{\text{th}}$  grid, where  $i \in N$ , is chosen randomly. Based on the parameter  $s_i$ , the customer  $n$  generates its utility function,  $U_n$ . Accordingly the customer chooses its service providing grid  $k \in N$ . It may happen that  $i \neq k$ . Figure 5 shows how the customers change their service providing grid and get better facilities.

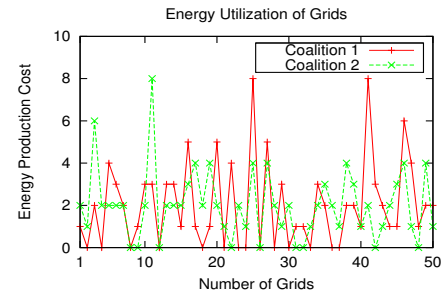


Fig. 5: Utilization of Energy

In Figure 5, the average energy distribution cost per unit for all the grids is less in Coalition 2 than Coalition 1. By varying the number of grids and the number of customers, we have shown in Figure 6 that the energy production cost of grids



change. In both the Figures, the grids will have much higher revenue and satisfaction parameter.

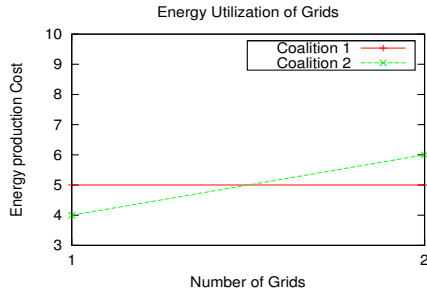


Fig. 6: Utilization of Energy

Due to random deployment, the grids and the customers are not uniformly distributed. The abrupt change in energy production cost and quality of energy service is due to the randomness of the grids and the customers in Figures 5 and 7.

### C. Quality of Energy Service

Figure 7 shows how the Quality of Service (QoS) for the customers can be improved by using this dynamic coalition formation scheme. In Figure 7, the reliability and the amount of energy distributed to the customers is obtained to be much more higher in the dynamically formed coalition, Coalition 2, than Coalition 1. So, it can be inferred that the QoS of Coalition 2 is much higher than the QoS of Coalition 1 in Figure 7.

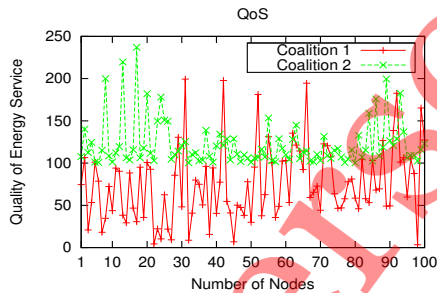


Fig. 7: Quality of Service

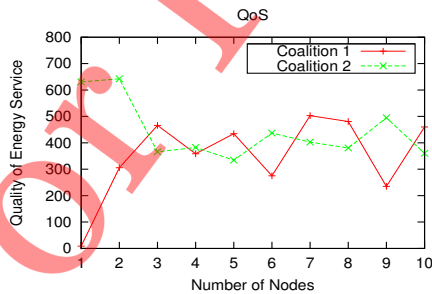


Fig. 8: Quality of Service

In Figure 8, we showed how the QoS changes with the change in the number of customers, and the number of grids. The QoS in Coalition 2 is much better than the QoS in Coalition 1.

## VI. CONCLUSION

In this paper, we formulated an MDP-based approach to study the problem of optimum energy distribution between the customers of the micro-grids. Based on this optimization method, we showed how the coalitions can be formed, and energy can be properly utilized. The simulation results show that the proposed approach yields improved results. Future extension of this work includes understanding how the coalition can be formed in a more optimal way, so that the services provided by the grids to the customers can be improved, thereby yielding utilization of smart grids.

## REFERENCES

- [1] S. Misra, P. Krishna, V. Saritha, and M. Obaidat, "Learning automata as a utility for power management in smart grids," *IEEE Communications Magazine*, vol. 51, no. 1, pp. 98–104, 2013.
- [2] W. Saad, Z. Hun, H. V. Poor, and T. Baser, "Game-theoretic methods for the smart grid: An overview of microgrid systems, demand-side management, and smart grid communication," *IEEE Signal Processing Magazine*, vol. 29, no. 5, pp. 86–105, September 2012.
- [3] W. Saad, Z. Han, and H. V. Poor, "Coalition game theory for cooperative micro-grid distribution networks," in *Proc. IEEE International Conf. on Comm.*, Kyoto, Japan, May 2011.
- [4] S. Misra, S. Mohan, and R. Choudhuri, "A probabilistic approach to minimize the conjunctive costs of node replacement and performance loss in the management of wireless sensor networks," *IEEE Transactions on Network and Service Management*, vol. 7, no. 2, pp. 107–117, 2010.
- [5] S. Misra and A. Jain, "Policy controlled self-configuration in unattended wireless sensor networks," *Journal of Network and Computer Applications*, vol. 34, no. 5, pp. 1530–1544, 2011.
- [6] S. Misra and S. Singh, "Localized policy-based target tracking using wireless sensor networks," *ACM Transactions on Sensor Networks*, vol. 8, no. 3, 2012.
- [7] S. D. Ramchurn, P. Vytelingum, A. Rogers, and N. Jennings, "Agent-based control for decentralised demand side management in the smart grid," *Proceedings of the Tenth International Conference on Autonomous Agents and Multiagent Systems (AAMAS 2011)*, no. 5-12, May 2011.
- [8] Z. Han and H. V. Poor, "Coalition game with cooperative transmission: a cure for the curse of boundary nodes in selfish packet-forwarding wireless networks," *IEEE Transactions on Comm.*, vol. 57, no. 1, pp. 203–213, January 2009.
- [9] V. C. Gungor and F. C. Lambert, "A survey on communication networks for electrical system automation," *Computer Networks*, vol. 50, pp. 877–897, January 2006.
- [10] P. Vytelingum, T. D. Voice, S. D. Ramchurn, A. Rogers, and N. R. Jennings, "Agent-based micro-storage management for the smart grid," *Proc. of the 9th International Conference on Autonomous Agents and Multiagent Systems*, vol. 1, 2010.
- [11] A. Molderink, V. Bakker, M. Bosman, J. Hurink, and G. Smit, "Demand side load management using a three step optimization methodology," *First IEEE International Conf. on Smart Grid Comm.*, pp. 431–436, October 2010.
- [12] —, "Management and control of domestic smart grid technology," *IEEE Transactions on Smart Grid*, vol. 1, no. 2, pp. 109–119, September 2010.
- [13] W. W. Weaver and P. T. Krein, "Game-theoretic control of small scale power systems," *IEEE Transactions on Power Delivery*, vol. 24, no. 3, pp. 1560–1567, July 2009.
- [14] C. Ibars, M. Navarro, and L. Giupponi, "Distributed demand management in smart grid with a congestion game," in *First IEEE Smart Grid Comm.*, October 2010, pp. 495–500.
- [15] J. N. Webb, *Game Theory Decisions, Interaction and Evolution*. Springer, 2006.