Introduction to Astronomy
AA 201
Fall Semester 2019

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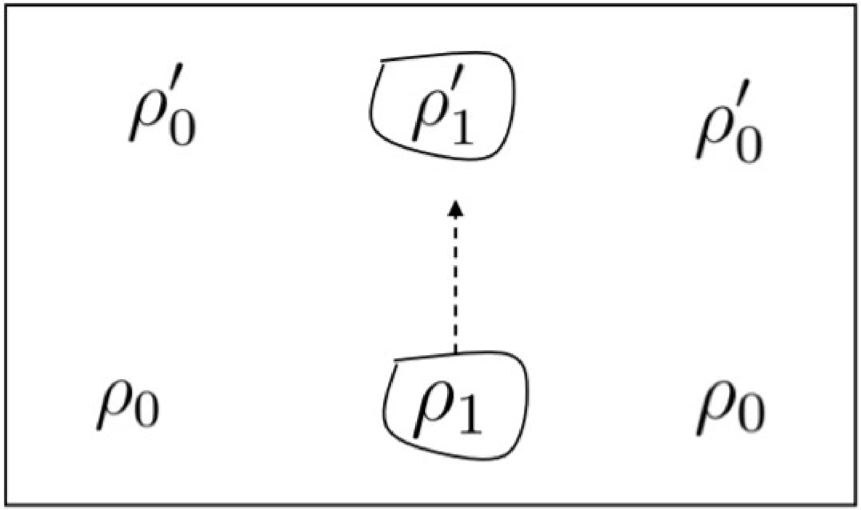
Course webpage:
http://www.iiti.ac.in/people/~manoneeta/courses/AA201_2019/



Assume ρ_1 is just slightly less than ρ_0

Gas element rises by buoyancy

-> new density is

$$\rho_1'$$

If $\rho_1' < \rho_0'$, it will keep rising, thus being unstable. Now, let's assume pressure equilibrium, so that $P'_1 = P_0'$. Then $T_1' > T_0'$, so since the underdense pocket rises, heat is transported upwards, as long as it doesn't start transferring heat to its surroundings.

So condition for gas to keep rising is:

$$\frac{d\rho_1}{dr} < \frac{d\rho_0}{dr} \quad (\text{both} \quad < 0)$$

If adiabatic (rises without transferring heat to surroundings), the adiabatic gas law for an ideal gas is obeyed:

$$P_1 V_1^{\gamma} = Const$$

(what should γ be in a stellar interior?)

or

$$P_1 = k\rho_1^{\gamma}$$

$$\text{Then } \frac{dP_1}{dr} = k\gamma\rho_1^{\gamma-1}\frac{d\rho_1}{dr} = k\gamma\frac{\rho_1^{\gamma}}{\rho_1}\frac{d\rho_1}{dr} = \gamma\frac{P_1}{\rho_1}\frac{d\rho_1}{dr}$$

$$\text{So } \frac{d\rho_1}{dr} = \frac{1}{\gamma}\frac{\rho_1}{P_1}\frac{dP_1}{dr} < \frac{d\rho_0}{dr} \qquad (1)$$

We still have the ideal gas law: $$P = \frac{\rho k T}{\mu m_H}$$

So we'll write: $$\frac{dP_0}{dr} = \frac{P_0}{\rho_0}\frac{d\rho_0}{dr} + \frac{P_0}{T_0}\frac{dT_0}{dr}$$

Solve for $d\rho_0/dr$, substitute in (1). Assuming pressure equilibrium,

$$P_1 = P_0, \frac{dP_1}{dr} = \frac{dP_0}{dr}$$

and now assume that the initial densities were nearly equal, so approximately $\rho_1 = \rho_0$.

Then
$$\frac{dT_0}{dr} < \left(1 - \frac{1}{\gamma}\right)\frac{T_0}{P_0}\frac{dP_0}{dr}$$

But $\frac{dP_0}{dr} = -\rho_0 g = -\rho_0 \frac{GM_r}{r^2}$ and $\frac{T_0}{P_0} = \frac{\mu m_H}{\rho_0 k}$

$$\boxed{\frac{dT_0}{dr} < -\left(1 - \frac{1}{\gamma}\right)\frac{\mu m_H}{k}\frac{GM_r}{r^2} = \frac{dT}{dr}\Big|_{ad}}$$

condition for convection

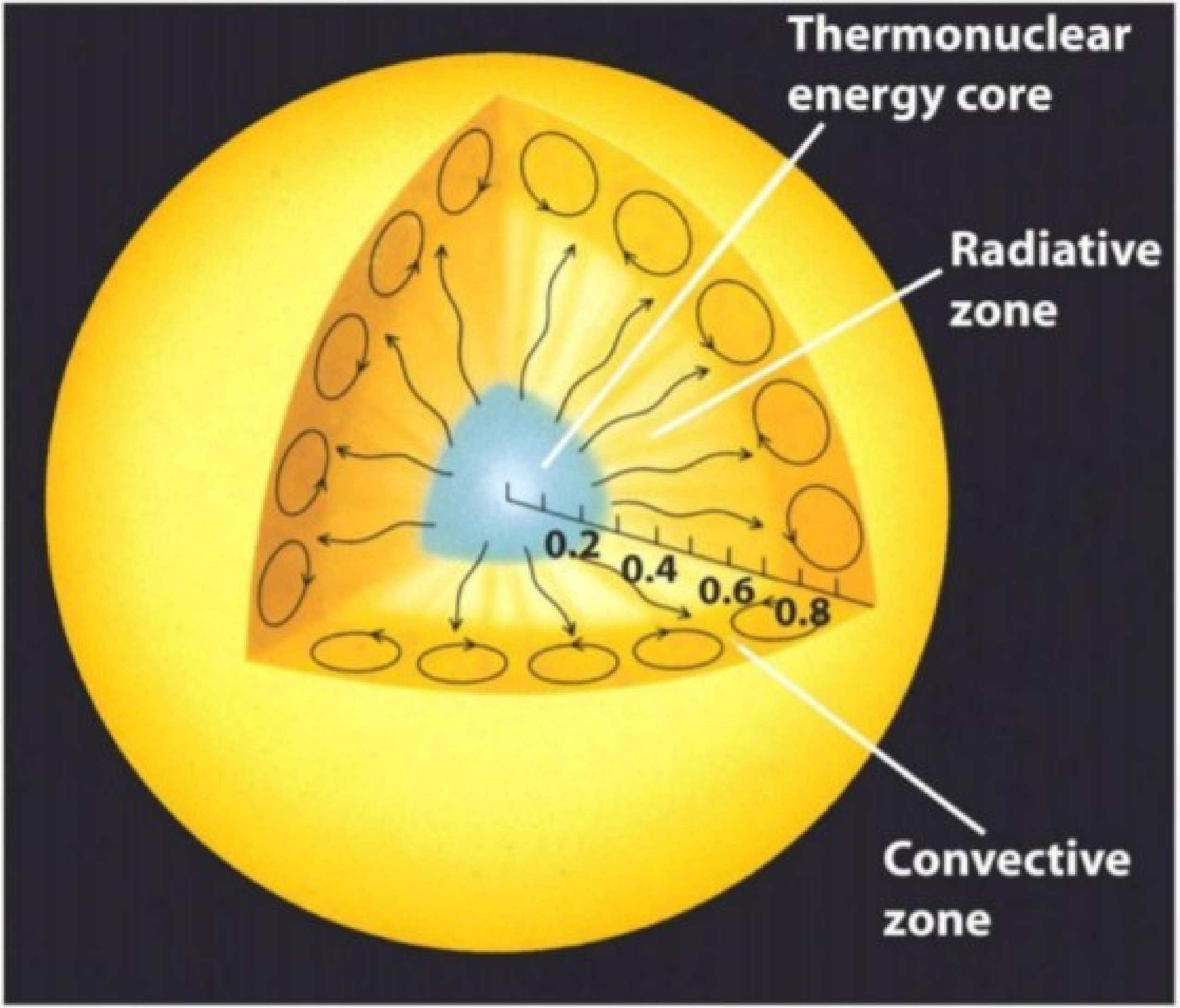
(Remember, $dT/dr < 0$)

Since the temperature gradients are negative, then if $|\frac{dT}{dr}| > |\frac{dT}{dr}|_{ad}$ bubbles will rise, and heat will be carried by convection.

Stops when bubble can finally cool (i.e. is no longer adiabatic), e.g. near the top of the Sun's atmosphere.

With the convection condition in this form, we can see when convection is likely, namely when

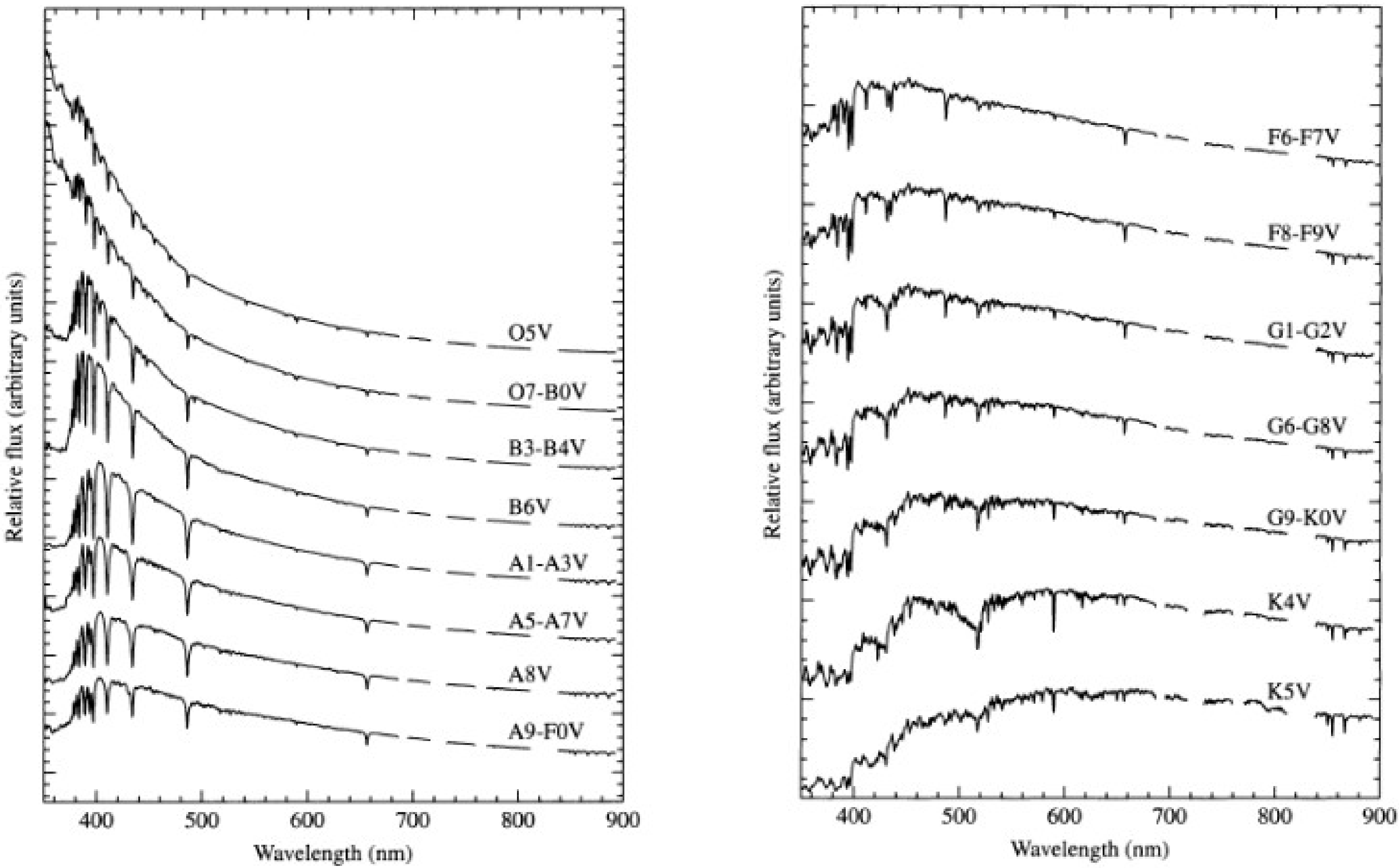
1. $\overline{\kappa}$ is large (so $|\frac{dT}{dr}|$ is large)
2. $\frac{L_r}{4\pi r^2}$ is high, deep in cores of massive stars (so $|\frac{dT}{dr}|$ is large)
3. $g = \frac{GM_r}{r^2}$ is low (so $|\frac{dT}{dr}|_{ad}$ is low)


Thermonuclear energy core
Radiative zone
Convective zone
0.2
0.4
0.6
0.8

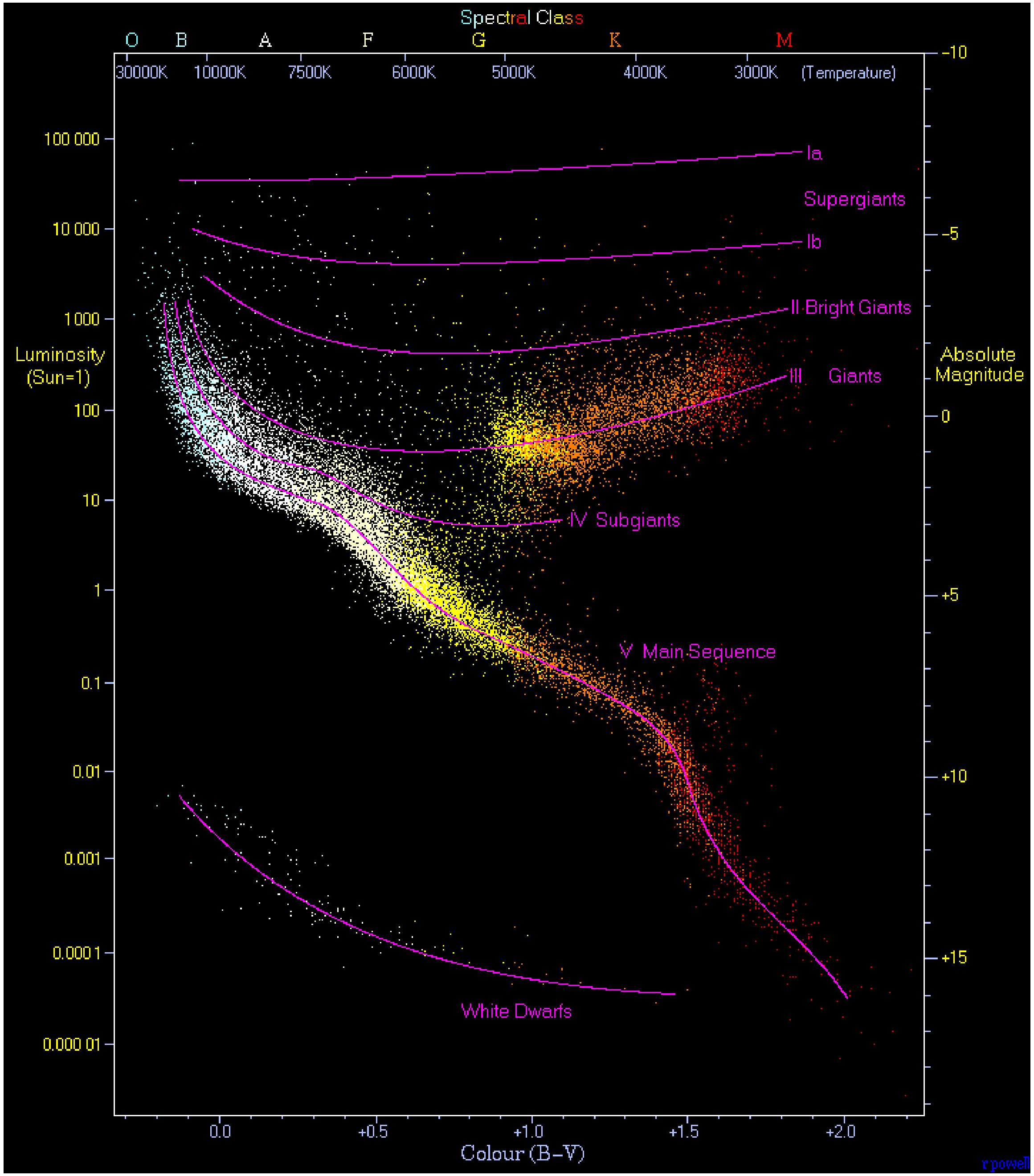
Stellar Spectral Classification Scheme

Spectral Type	Characteristics
O	Hottest blue-white stars with few lines Strong He II absorption (sometimes emission) lines. He I absorption lines becoming stronger.
B	Hot blue-white He I absorption lines strongest at B2. H I (Balmer) absorption lines becoming stronger.
A	White Balmer absorption lines strongest at A0, becoming weaker later. Ca II absorption lines becoming stronger.
F	Yellow-white Ca II lines continue to strengthen as Balmer lines continue to weaken. Neutral metal absorption lines (Fe I, Cr I).
G	Yellow Solar-type spectra. Ca II lines continue becoming stronger. Fe I, other neutral metal lines becoming stronger.
K	Cool orange Ca II H and K lines strongest at K0, becoming weaker later. Spectra dominated by metal absorption lines.
M	Cool red Spectra dominated by molecular absorption bands, especially titanium oxide (TiO) and vanadium oxide (VO). Neutral metal absorption lines remain strong.
L	Very cool, dark red Stronger in infrared than visible. Strong molecular absorption bands of metal hydrides (CrH, FeH), water (H_2O), carbon monoxide (CO), and alkali metals (Na, K, Rb, Cs). TiO and VO are weakening.
T	Coolest, Infrared Strong methane (CH_4) bands but weakening CO bands.

Stellar Spectral Classification Scheme

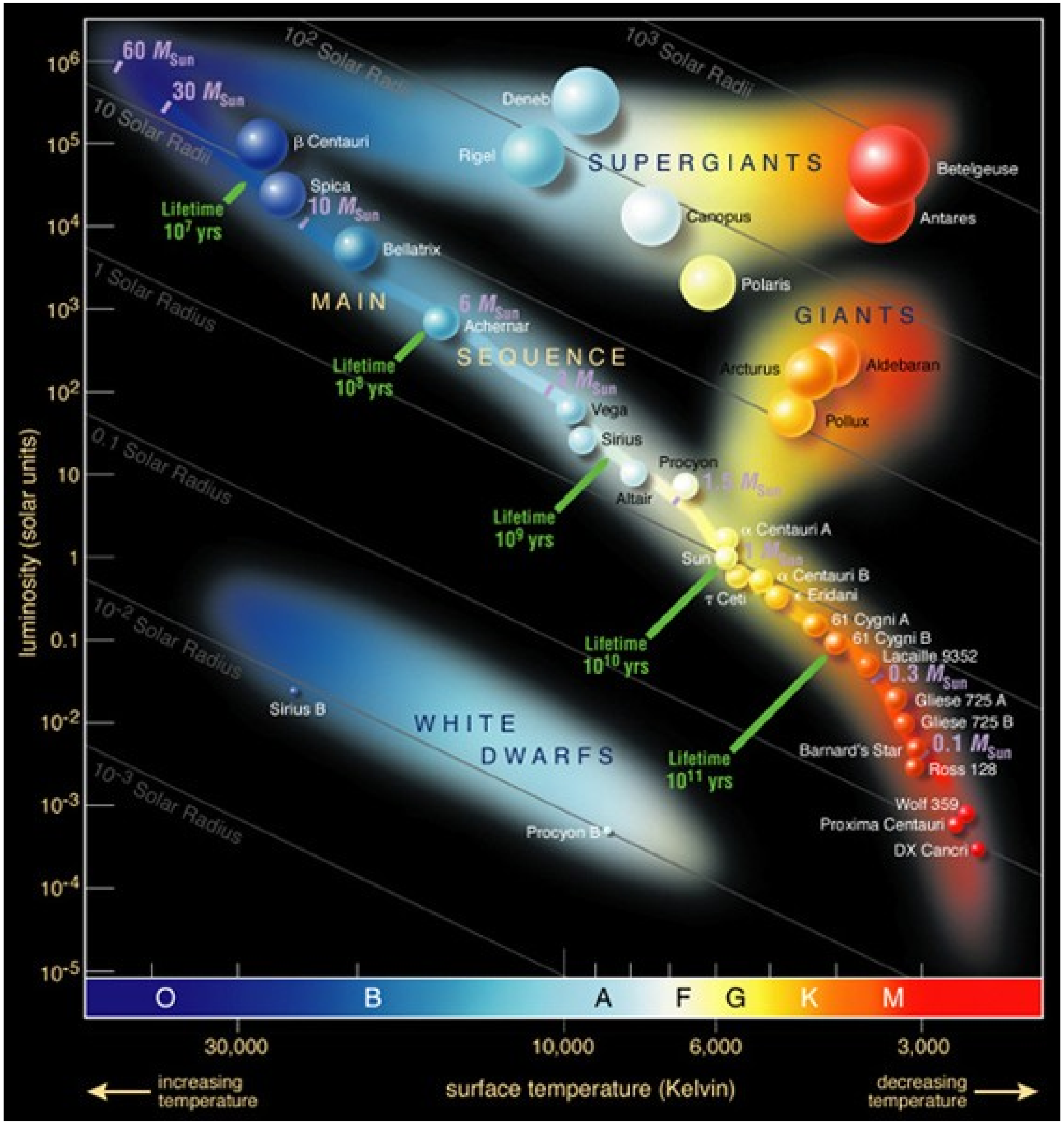


H-R diagram

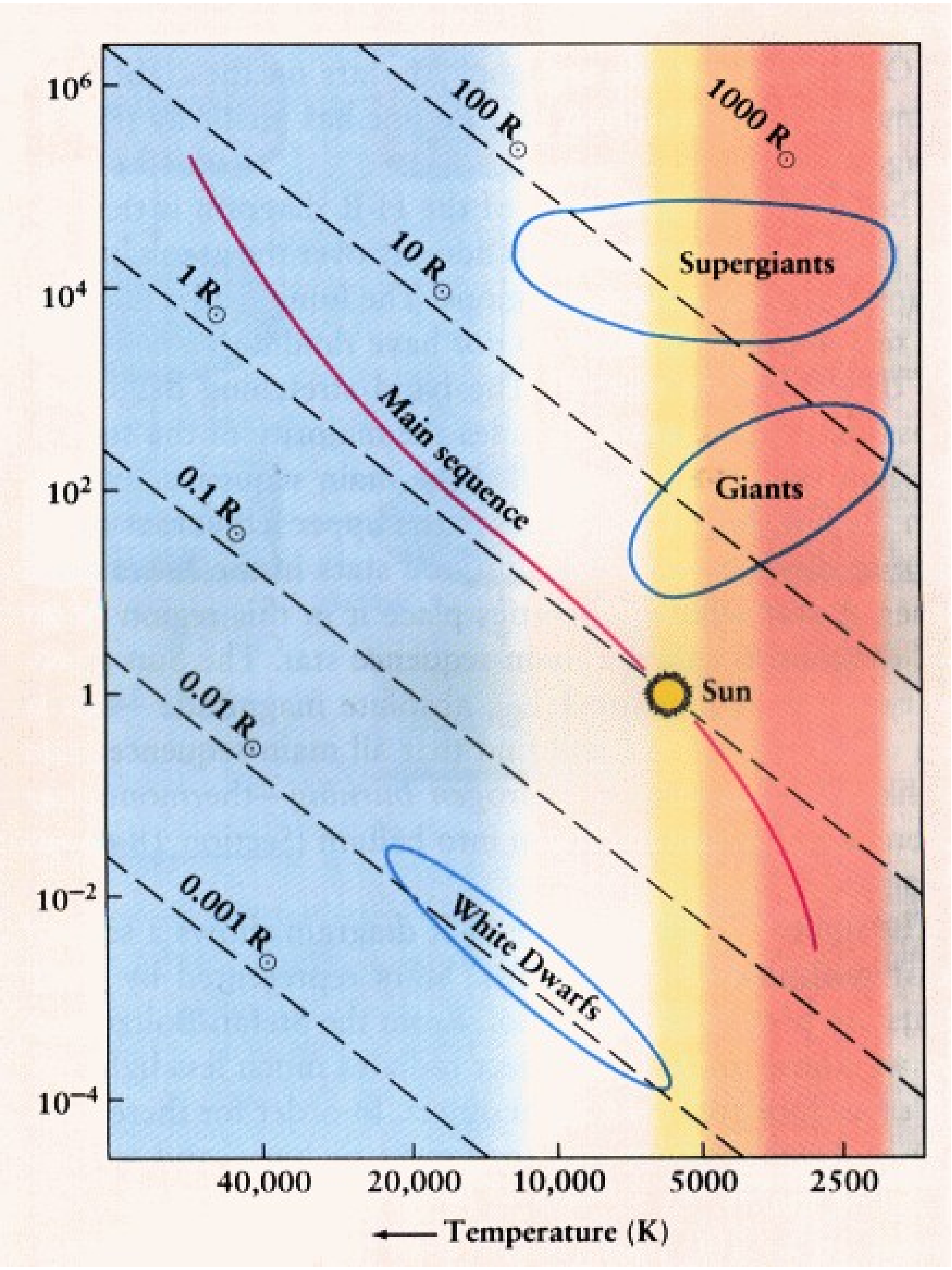


Spectral Class
O B A F G K M
30000K 10000K 7500K 6000K 5000K 4000K 3000K (Temperature)
Luminosity (Sun=1)
100 000
10 000
1000
100
10
1
0.1
0.01
0.001
0.0001
0.000 01
Absolute Magnitude
-10
-5
0
+5
+10
+15
Ia
Supergiants
Ib
II Bright Giants
III Giants
IV Subgiants
V Main Sequence
White Dwarfs
0.0
+0.5
+1.0
+1.5
+2.0
Colour (B-V)
r.powell

H-R diagram

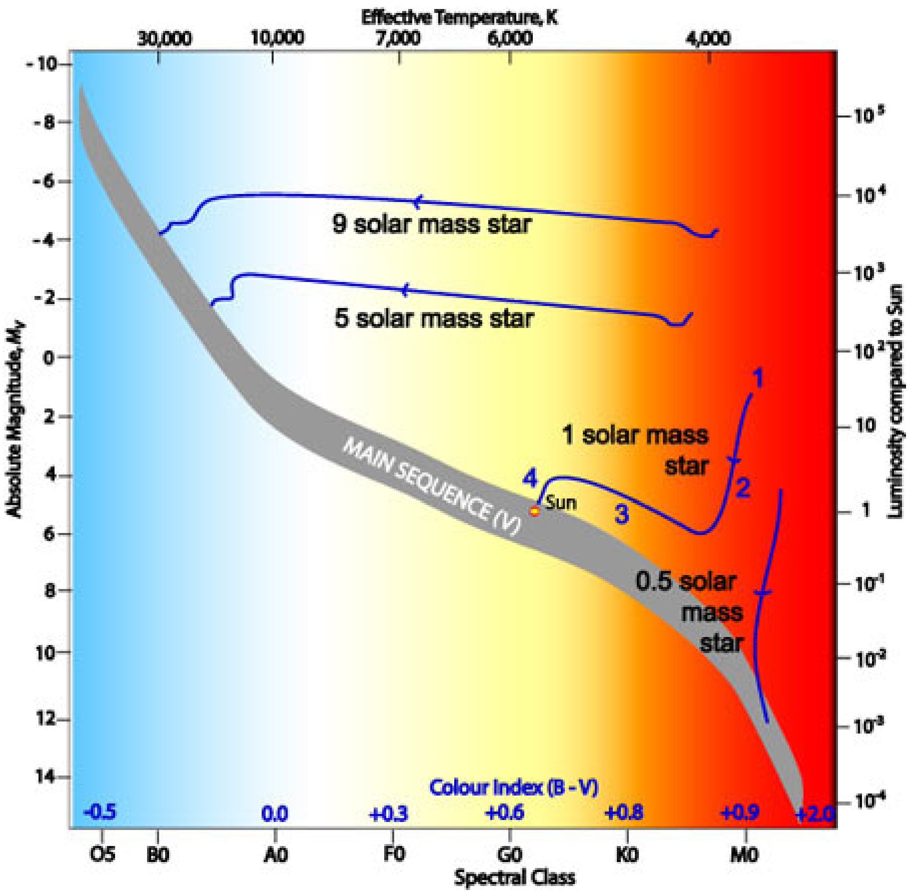


luminosity (solar units)
surface temperature (Kelvin)
increasing temperature
decreasing temperature
SUPERGIANTS
GIANTS
MAIN SEQUENCE
WHITE DWARFS
O B A F G K M
30,000
10,000
6,000
3,000
Deneb
Rigel
Canopus
Betelgeuse
Antares
Polaris
Arcturus
Aldebaran
Pollux
β Centauri
Spica
Bellatrix
Achernar
Vega
Sirius
Procyon
Altair
α Centauri A
Sun
α Centauri B
τ Ceti
ε Eridani
61 Cygni A
61 Cygni B
Lacaille 9352
Gliese 725 A
Gliese 725 B
Barnard's Star
Ross 128
Wolf 359
Proxima Centauri
DX Cancri
Sirius B
Procyon B

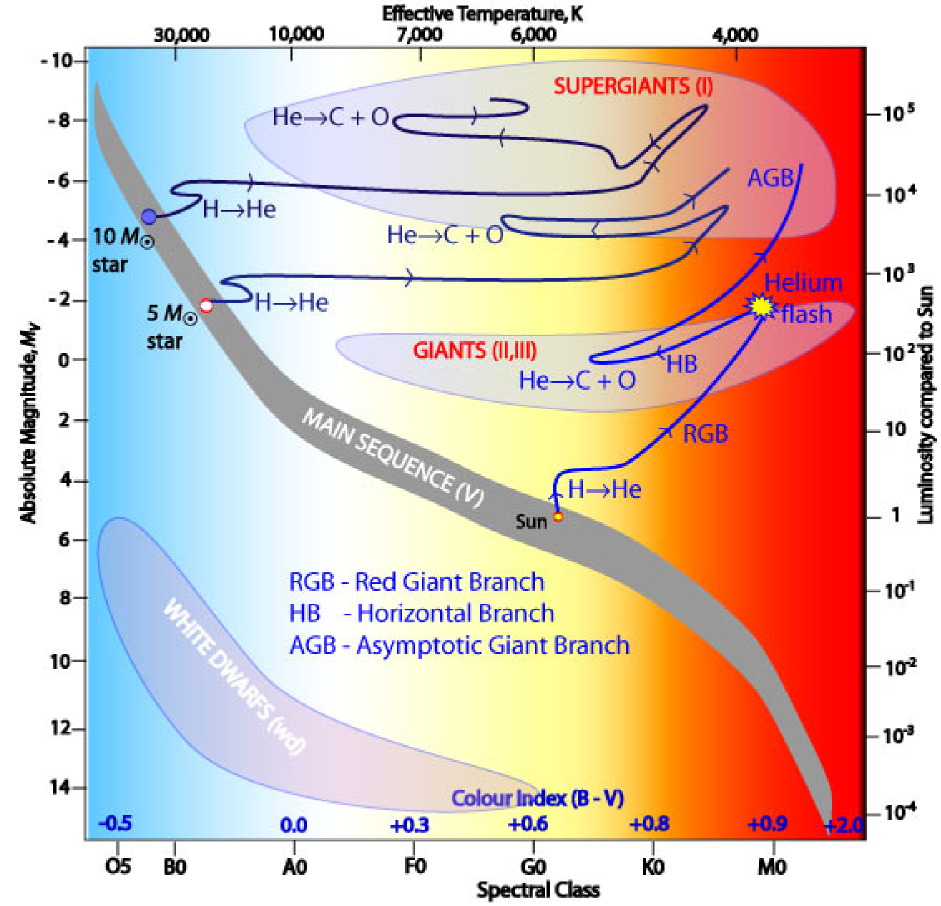


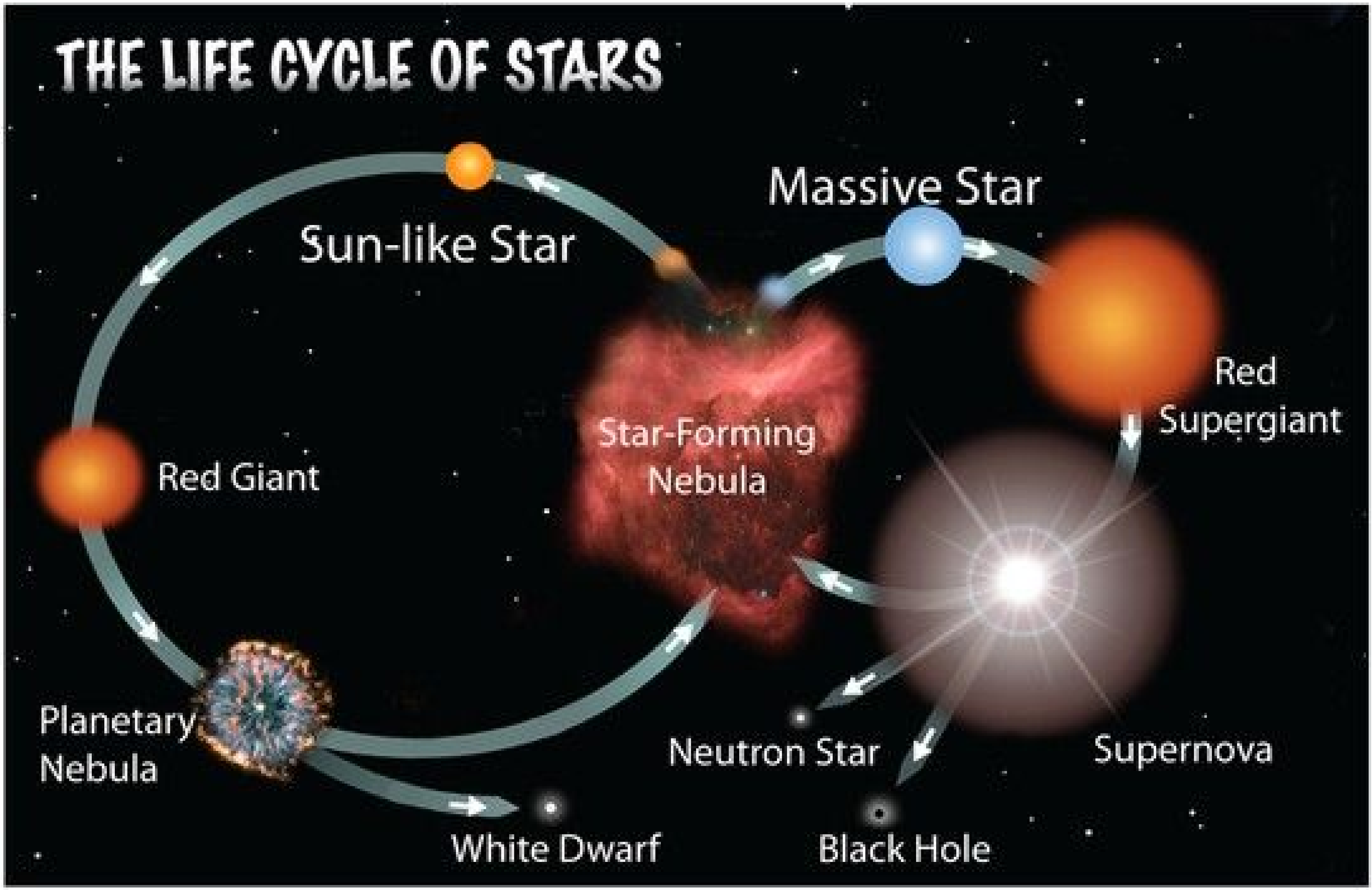
10^6
10^4
10^2
1
10^−2
10^−4
100 R☉
1000 R☉
10 R☉
1 R☉
0.1 R☉
0.01 R☉
0.001 R☉
Supergiants
Giants
Main sequence
Sun
White Dwarfs
40,000
20,000
10,000
5000
2500
← Temperature (K)

Theoretical Hayashi Tracks of Protostars



Evolutionary Tracks off the Main Sequence




THE LIFE CYCLE OF STARS
Sun-like Star
Massive Star
Red Supergiant
Star-Forming Nebula
Red Giant
Supernova
Planetary Nebula
Neutron Star
White Dwarf
Black Hole