

AA 472/672 : Galactic and Extra-Galactic Astronomy - Tutorial

Questions 1 and 2 have to be submitted in class

January 10, 2019

1. *Potential-Density Pair:* In a spherical galaxy, the density of matter varies with radius as -

$$\rho(r) = \frac{M}{4\pi} \frac{a}{r^2(r+a)^2} \quad (1)$$

where M and a are constants

- Estimate the total mass of the galaxy
- Obtain the expression for gravitational potential $\Phi(r)$.
- Show that for large radii $r \gg a$, the potential approaches that of a point mass.
- Find the expression for the circular velocity and show that it is approximately constant as small radii ($r \ll a$) and find its radial dependence at large radii ($r \gg a$).

2. *Effective Potential and Orbits:*

- For any general inverse-square law force, write the expression for the effective potential. Sketch the curve of the effective potential with radius and convince yourself that there exists a minimum value of potential V_{\min} . Assume a particle with unit mass and energy E , give physical insight into what happens to the particle motion when a) $E \gg 0$, b) $E < V_{\min}$ c) $E = V_{\min}$ and d) $0 < E < V_{\min}$
- Repeat the above and sketch the figure for force $F = -kx^{-4}$ and find the different kinds of orbits possible for all possible ranges of values of E and r .

3. *Extra question home work:* What is *Bertrand's theorem* and give the proof of the same. Convince yourself that there are only two central forces - a) inverse square law force $F = -k/r^2$ and b) isotropic harmonic oscillator force $F = -kr$ in which closed repeated orbits are produced.