GOOD EVENING

Outine

- Introduction
- Engines-Efficiency
- Laws of Thermodynamics
- Maxwell's Demon can violate the laws of Thermodynamics
- Szilard's resolution, Information
- Landauer's Erasure Principle
- Quantum Information

Introduction

- In the beginning of nineteenth century, engineers were trying to improve the efficiency of thermal engines by various means.
- Sadi Carnot (1796-1832) found the maximum energy that can be extracted from thermal machines irrespective of the substance used.
- He invented an ideal engine now called the Carnot's Engine and other machines could at best equal it's efficiency.

Introduction

- This led to the second law of thermodynamics, which can be stated in several forms (Lord Kelvin(Thomson), Rudolf Clausius).
- Here we will use the statement that **Heat energy cannot be completely converted to mechanical energy just from one heat source (reservoir)**
- Conservation of energy says you can break even.
- Second law says you will always lose (when trying to convert heat to work)

Introduction

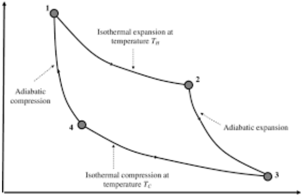
- In 1867, James Clerk Maxwell suggested a hypothetical method to violate second law by an "intelligent person" who has been nicknamed
- **Maxwell's Demon**

Introduction

- Addressing this issue has led to radically new view of information and has led to obtain a quantification of information.
- This talk will give an overview of these interesting and useful developments in the last few decades

Carnot's Engine

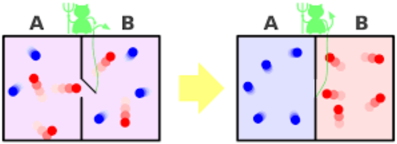
Carnot's engine is shown in the diagram between (V) Volume on the x-axis and (P) Pressure on the y-axis of the system and is shown below.



Maxwell's Demon

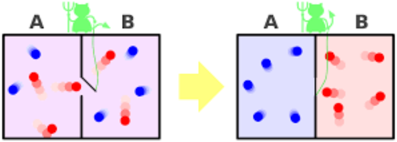
- Consider a box of ideal gas (gas having small density and at high temperature is an ideal gas). The molecules in the box execute random motion and they collide with walls confining them.
- This gives rise to pressure of the gas (due to walls exerting an opposite force on the molecules).
- Insert a partition in the middle separating it into two equal sides (Left and Right)

Maxwell's demon



- Initially, there are molecules of different speeds moving randomly in both sides of the box A and B.
- The one's with higher speed is represented by red and the slower one's by blue.

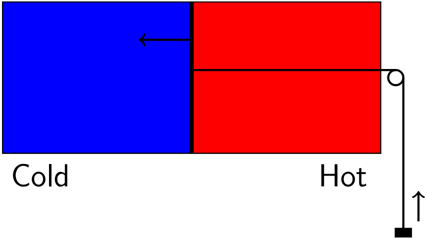
Maxwell's demon



- The Maxwell Demon, earmarks the red ones and allows them to move to the right side (B) by opening a trap in the partition, when they are heading that way.
- Similarly he allows the slower one's to move to the left.
- This leads to the situation shown on the right. Since the right side B has more energetic particles they are at a higher temperature and pressure than the side A.

Maxwell's demon

- Heat converted to work with 100% efficiency, by allowing the partition to move.
- It will move to left and lift the weight. Violates the second law of thermodynamics

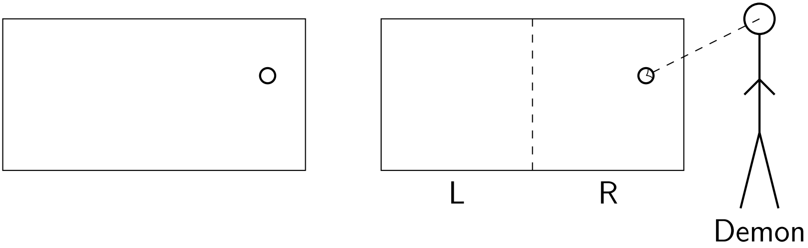


Szilard's analysis

- Leo Szilard offered a resolution to the problem of Maxwell's demon discussed in the previous section by considering heat engine with a system consisting of one molecule in a box .
- The box is at thermal equilibrium with a reservoir at temperature T.
- The argument which will be outlined below led him to the conclusion - to quote his words,
- One may reasonably assume that a measurement procedure is fundamentally associated with a certain definite entropy reduction, and that this restores concordance with the second law.

Szilard's analysis

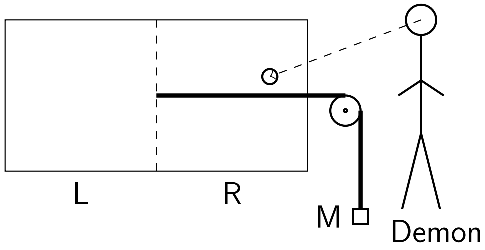
- Szilard's one molecule engine is shown in the figure . The reservoir keeping it a temperature T is not shown.
- The box of volume V contains one molecule .
- Next a thin massless thermally insulating partition is inserted dividing the box into two equal parts each having a volume $V/2$. This is shown below.

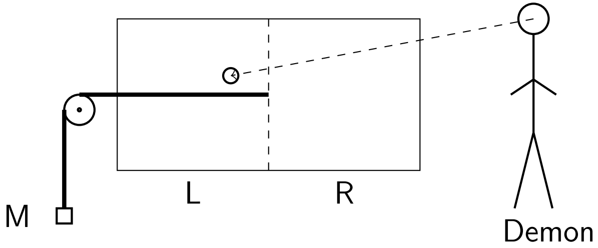


Szilard's analysis

- The demon observes the molecule and the measurement tells him in which side of the box is the molecule.
- If the molecule is on the right side as shown, he attaches a rope to the partition, the other end of which goes over a massless, frictionless pulley and is attached to a mass M as shown in the figure.
- The partition is free to slide without friction inside the box.

Szilard's analysis




L
R
M
Demon

Szilard' analysis

- Szilard conjectured that knowing which side of the box molecule is can not be obtained free of energy cost.
- He assumed if the energy spent in obtaining the information should match the energy obtained from the above method.

Szilard's analysis

- It is easy to calculate that quantity and we state the answer as the energy required in acquiring the information is

$$k_B \, T \ln 2$$

  where k_B is a constant of proportionality, the Boltzmann constant, T the temperature and $\ln$ is the natural logarithm.
- So though we gained energy from the system, an equal amount of energy was spent in acquiring the information to make the system deliver.

Landauer's analysis

- Landauer noticed a certain lacuna in the argument of Szilard
- The demon(memory state) possesed the information in the final state, whereas initially he did not possess it.
- Inorder to complete a cycle and start all over again, the memory state must be restored to the initial value.
- This means that the information stored in the memory must be erased.

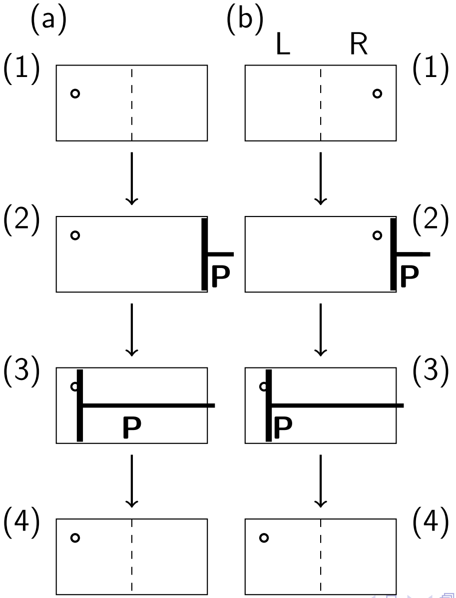
Landauer's principle

- Further, Landauer could construct a physical process by which information could be stored without expending energy, that is the information could be stored without performing work on the system.
- The quantitative statement is that the minimum energy required to erase one bit (0,or 1) is

$$k_B T \ln(2)$$

  where T is the temperature. We will see how this comes about in the next slide
- Several experiments have been carried out in the last decade and have found agreement with the principle.

Landauer's principle



Landauers principle

- The steps for erasure in the memory are the following. We will the molecule to a standard memory state, irrespective of its original position, say to the left of the box.
- This way, we do not know where the molecule was before erasure.
- This is done by removing the partition and pushing the molecule to the left of the box, thus erasing the information.
- However during pushing the molecule, one has to perform work and this exactly equals the work extracted from the engine.Thus the second law is restored.

Landauer's principle

- The works of Szilard and Landauer have brought a new concept that information has to be treated as a physical quantity and is debated in some quarters.
- However, the concept has been useful in understanding several results leading to information exchange, much like the laws of thermodynamics has been successful in understanding heat exchange.

Information and quantum mechanics

- With modern technology, one can construct very small systems which obey the rules of quantum mechanics and so considerable work has been done in the last three decades based on analysing the above ideas quantum mechanically.
- This goes under the name of quantum information and quantum computation.
- I will take one example in the form of a game to show how the information generated in using a quantum system is more efficient than using classical systems.

Information and quantum mechanics

- A classical information can be thought of as being coded in bits (which take the value 0 or 1).
- Thus 5 bits is enough to make 32 pieces of information.
- All the present day computers where information processing takes place use bits as basic units.
- In contrast quantum systems can use quantum bits which is commonly referred to qubits. We discuss how a qubit functions in the next slide

Qubit

- A qubit is written as a vector in 2 dimensions as

$$\begin{pmatrix} a \\ b \end{pmatrix} \tag{1}$$

  where a and b are complex numbers and it is normalized as $|a|^2 + |b|^2 = 1$
- Now the quantum system can take values different from just 1 or 0 , giving it enormously more scope for manipulation.
- We will use the notation

$$|0\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \qquad |1\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

Quantum operations on bits

- One can perform the operations using quantum operations on qubits:

  $$|a\rangle \rightarrow (-1)^a |a\rangle$$

  where $a = \pm 1$.
- This means when $a = 0$, the factor $(-1)^a = 1$ and when $a = 1$, $(-1)^a = -1$.
- Further we can use what is called a Hadamard transformation, which does the following:

$$|0\rangle \rightarrow \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle) \qquad |1\rangle \rightarrow \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle)$$

Information processing: classical vs quantum mechanics

- Consider the following game between three persons, say A,B and C.
- There are four apples.
- Some number of apples are (is) cut into half and each can be given a whole number , which can be 0,1,2,3 or 4 **or** half integer of apples which can be 1/2,3/2,5/2 or 7/2.

Information processing: classical vs quantum mechanics

- They can not communicate between themselves
- But B and C can communicate with another person D, who communicates there statement truthfully to A.
- A has to answer whether the total number of apples distributed to A,B and C is an even or odd number.
- They can have a predetermined strategy for communicating to A (through D) so that A can make a decision.

Information processing: classical vs quantum

- Our "classical" strategy could be the following.
- Notice we need to consider only the case with a total of two apples as increasing it to four is just an addition by 2 which will not change the even/odd property of the total sum.

Information processing: classical vs quantum

- Since A knows his/her apple, we need to consider the 16 possibilities of distribution amongst B and C.
- Half of the possibilities would be an integer and the other half would be half integer.
- A, knowing his/her number can immediately distinguish between the two possibilities

Infroamtion processing : calssical vs quantum

- Assume A has a whole number of apples.
- If A has 0; the (B,C) can have 6 possibilies :(0,0),(1/2,1/2),(1,0),(0,1),(3/2,1/2),(1/2,3/2).
- If A has 1; we have 4 possibilies for (B,C) : (0,0),(1,0),(0,1),(1/2,3/2);
- If A has $1/2$: then we have 4 possibilities of (B,C): (0,1/2),(1/2,0),(1/2,1),(1,1/2)
- and if A has $3/2$: then we have 2 possibilities for (B,C): (1/2,0),(0,1/2).
- The total number of possibilities is 16

Information processing: classical vs quantum

- Assume it was decided earlier that B will her/his flag raised up if the number is 1/2 or 0 and lowered down if it is 1 or 3/2
- C will have flag raised up when it is 1/2 or 1 and lowered down if it is 0 or 3/2

Information processing: classical vs quantum

- Then we can have the following possibilities
- If (B,C) is (0,0) flag is up and down; If it is (1,1) it is down and up. Both these give B+C as even.
- If (B,C) is (0,1), both flags will be up and if it is (1,0) both flags will be down.
- Thus A can decide whether the total number is even or odd because he/she knows his/her number of apples.

Information processing : classical vs quantum

- A similar analysis can be done when B and C each hold half-integer of apples
- It turns out A can make a correct decision.
- However, if A holds a half integer number of apples
- Now consider the case when (B,C) have (1/2,0) apple, in which case flags will be **up,down**
- This signal could also come when (B,C) is (0,3/2).
- This means A cannot make up her mind on the evenness of the total number of apples.
- In such cases, A has to guess and she/he would get it right only 50% of the time.

Information processing : classical vs quantum

- One can verify this is indeed the case for all the cases when A holds a half integer of apples.
- Thus this strategy gives the total chance as 75% .
- One can try other strategies and the maximum possibility is only 75%.

Information processing : classical vs quantum

- We start A,B and C possessing three qubits in the state

$$|\psi\rangle = \sqrt{1}2[|0\rangle_A|0\rangle_B|0\rangle_C + |1\rangle_A|1\rangle_B|1\rangle_C]$$

- Note such a state is superposition of two different states, which is not possible classically.
- A performs the following quantum operation on her/his qubit:

$$|0\rangle_A \rightarrow |0\rangle_A \qquad |1\rangle_A \rightarrow (-1)^{x_A}|1\rangle_A$$

where x_A is the number of apples A possesses.

Information processing : classical vs quantum

- B and C perform the same operation or their respective qubits. Thus the new state after the operations is performed by the three of them is

$$|\psi\rangle' = \sqrt{\frac{1}{2}}[(|0\rangle_A\,|0\rangle_B|0\rangle_C + (-1)^{x_A+x_B+x_C}|1\rangle_A|1\rangle_B|1\rangle_C]$$

$$= \sqrt{\frac{1}{2}}[(|0\rangle_A\,|0\rangle_B|0\rangle_C \pm |1\rangle_A|1\rangle_B|1\rangle_C]$$

depending on whether $x_A + x_B + x_C$ is even or odd.

Information processing : classical vs quantum

- Now each of them apply the Hadamard transformation

$$|0\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle) \qquad |1\rangle = \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle)$$

on their respecive qubits.

Information processing : classical vs quantum

- This makes the state

$$|\psi'\rangle_{\pm} = \left(\frac{|0\rangle_A + |1\rangle_A}{\sqrt{2}}\right)$$

$$\times \left(\frac{|0\rangle_B + |1\rangle_B}{\sqrt{2}}\right)\left(\frac{|0\rangle_C + |1\rangle_C}{\sqrt{2}}\right)$$

$$\pm \left(\frac{|0\rangle_A - |1\rangle_A}{\sqrt{2}}\right)\left(\frac{|0\rangle_B - |1\rangle_B}{\sqrt{2}}\right)\times\left(\frac{|0\rangle_C - |1\rangle_C}{\sqrt{2}}\right.$$

Information processing: classical vs quantum

- For $|\psi'\rangle_+$, this simplifies to

$$|\psi'\rangle_+ = \frac{1}{2}[|0\rangle_A|0\rangle_B|0\rangle_C + |0\rangle_A|1\rangle_B|1\rangle_C]$$

$$+ [|1\rangle_A|0\rangle_B|1\rangle_C + |1\rangle_A|1\rangle_B|0\rangle_C]$$

- They now use the pre- agreed convention that (B or C) will raise their flag up if their qubit is in the state 0 and down if it is in the state 1.
- A knowing her/his state immediately knows whether the number is even or odd.

Information processing: classical vs quantum

- Exactly the same logic works for the state $|\psi'\rangle_-$, which is

$$|\psi'\rangle_- = \frac{1}{2}\left[|0\rangle_A|0\rangle_B|1\rangle_C + |0\rangle_A|1\rangle_B|0\rangle_C\right]$$

$$+ \left[|1\rangle_A|0\rangle_B|0\rangle_C + |1\rangle_A|1\rangle_B|1\rangle_C\right]$$

- Thus, quantum processing gets the information 100% of the time

Summary

- Information has to treated physically and is a physical resource just like energy is
- Quantum processing of information can yield more than classical processing.
- The subject encompasses several areas : Quantum Thermodynamics, Open quantum system, Quantum cryptology to name a few.
- Exciting days lie ahead.

THANK YOU